

Volume of a hemisphere, sphere, cone and frustum

Solid	Volume	Curved surface	Total surface	Notes
Sphere, radius r	$\frac{4}{3} \pi r^3$	$4\pi r^2$	$4\pi r^2$	both on the GCSE sheet
Hemisphere, radius r	$\frac{2}{3} \pi r^3$	$2\pi r^2$	$3\pi r^2$ (with the flat face)	halve the sphere; add the circle if solid
Cone, radius r , height h , slant l	$\frac{1}{3} \pi r^2 h$	$\pi r l$	$\pi r l + \pi r^2$	$l = \sqrt{r^2 + h^2}$
Frustum, radii R and r , height h , slant l	$\frac{1}{3} \pi h (R^2 + Rr + r^2)$	$\pi (R + r) l$	$\pi (R + r) l + \pi R^2 + \pi r^2$	$l = \sqrt{h^2 + (R - r)^2}$; or big cone minus small cone
Cylinder, radius r , height h	$\pi r^2 h$	$2\pi r h$	$2\pi r h + 2\pi r^2$	the cone is a third of it

Where they come from

[] Sphere: two thirds of the cylinder that just contains it (Archimedes): $\frac{2}{3} \times \pi r^2 \times 2r = \frac{4}{3} \pi r^3$; its surface equals that cylinder's curved area $4\pi r^2$

[] Cone: a third of the cylinder with the same base and height (three cones of water fill it); the curved surface unrolls to a sector of radius l and arc $2\pi r$, area $\frac{1}{2} \times l \times 2\pi r = \pi r l$

[] Frustum: full cone minus the small cone removed. Small cone height x from similar triangles: $x \div r = (x + h) \div R$, so $x = hr \div (R - r)$. Show this step if you subtract

Worked

Solid	Working	Answer
Sphere $r = 6$	$\frac{4}{3} \times \pi \times 216$; surface $4\pi \times 36$	$288\pi = 905 \text{ cm}^3$; $144\pi = 452 \text{ cm}^2$
Solid hemisphere $r = 6$	$\frac{2}{3} \times \pi \times 216$; $3\pi \times 36$	$144\pi = 452 \text{ cm}^3$; $108\pi = 339 \text{ cm}^2$
Cone $r = 5$, $h = 12$	$\frac{1}{3} \times \pi \times 25 \times 12$; $l = 13$; $\pi \times 5 \times 13 + 25\pi$	$100\pi = 314 \text{ cm}^3$; $90\pi = 283 \text{ cm}^2$
Frustum $R = 15$, $r = 10$, $h = 24$ (a bucket)	$\frac{1}{3} \times \pi \times 24 \times (225 + 150 + 100)$	$3,800\pi = 11,900 \text{ cm}^3 = 11.9 \text{ L}$
Same, by subtraction	$x = 48$; $5,400\pi - 1,600\pi$	$3,800\pi$
Hemisphere on a cylinder, $r = 3$, $h = 8$ (silo)	$72\pi + 18\pi$	$90\pi = 283 \text{ m}^3$
Cone on a hemisphere, $r = 3$, cone $h = 10$ (ice cream)	$30\pi + 18\pi$	$48\pi = 151 \text{ cm}^3$
Sphere of volume 36π : radius	$r^3 = 3 \times 36\pi \div (4\pi) = 27$	3 cm

From

sciencewithshobha.com/blog/volume-of-a-hemisphere-sphere-cone-and-frustum-the-formulas-where-they-come-from-surface-areas-worked-examples-and-practi

ce/ · 1-on-1 online maths, KS2 to A level, Grades 3 to 12, free 30-minute first class.

Composite solids, the mistakes, and a practice set

- [] Volume: add the parts (subtract a hollow). Surface area: add only the EXPOSED faces. Hemisphere on a cylinder: $2\pi r^2 + 2\pi rh + \pi r^2$ (the join hides two circles). Cone on a hemisphere: $\pi rl + 2\pi r^2$
- [] Similar solids: lengths $\times k$, areas $\times k^2$, volumes $\times k^3$. A frustum with two radii is a similar-triangles question
- [] Units: 1 litre = 1,000 cm³; 1 m³ = 1,000 litres
- [] Given on recent GCSE sheets: sphere volume and surface, cone volume and curved surface. Derive: hemisphere, frustum, slant height. Check your year's arrangements. CBSE expects every formula from memory

Mistakes

- [] Diameter as r ; hemisphere treated as a sphere; the flat face forgotten on a solid hemisphere
- [] Slant height in $\frac{1}{2}\pi r^2 h$, or height in πrl ; $(\pi r)^3$ instead of πr^3
- [] Frustum: the wrong height (it wants the frustum's own); hidden faces added in a composite surface area
- [] Square root instead of cube root when finding r from a volume; π rounded early

Curriculum	Where
England	GCSE Higher spheres, cones, frustums, composite and similar solids; Foundation with formulas given
Australia	Year 10 volumes of cones and spheres; 10A composites and frustums
CBSE	Class 9 Surface Areas and Volumes; Class 10 combinations (frustum in many state boards)
US	Geometry HSG-GMD.A.1 and A.3: Cavalieri, formulas and problems

Practice

1. Volume and total surface area of a solid hemisphere of radius 9 cm, in terms of π .
2. A cone has radius 8 cm and height 15 cm. Volume and total surface area to 3 s.f.
3. A frustum has radii 12 cm and 6 cm and height 8 cm. Volume? Height of the small cone removed?
4. A toy is a cone of height 7 cm on a hemisphere of radius 3 cm. Total volume to 3 s.f.
5. A sphere has volume 972π cm³. Its radius?
6. A hemispherical bowl of inner radius 10 cm, full of water, is emptied into a cylinder of radius 5 cm. Depth of water?

Answers

1. 486π cm³; 243π cm². 2. $320\pi = 1,010$ cm³; $l = 17$, $136\pi + 64\pi = 200\pi = 628$ cm². 3. $\frac{1}{3} \times \pi \times 8 \times (144 + 72 + 36) = 672\pi = 2,110$ cm³; $x \div 6 = (x + 8) \div 12$, $x = 8$ cm. 4. $21\pi + 18\pi = 39\pi = 123$ cm³. 5. $r^3 = 729$, $r = 9$ cm. 6. $\frac{2}{3} \times \pi \times 1,000 = 25\pi \times h$, $h = 26.7$ cm.