

Types of standard deviation: population vs sample, the formulas, and one worked example done both ways

The two formulas

	Population standard deviation (σ , sigma)	Sample standard deviation (s)
Use when	You have every value in the group you care about (a whole class, every product made today)	Your data is a sample and you want to estimate the spread of the larger population
Formula	$\sigma = \sqrt{[\sum (x - \mu)^2 / N]}$	$s = \sqrt{[\sum (x - \bar{x})^2 / (n - 1)]}$
Divide by	N, the number of values	n - 1 (Bessel's correction: a sample underestimates spread, and n - 1 corrects it)
Mean symbol	μ (mu)	$\bar{x}$ (x-bar)
Excel / Sheets	=STDEV.P(range)	=STDEV.S(range) (older: =STDEV)
Calculator	σx or σn	$s x$ or $\sigma n-1$

The steps

1. Find the mean. 2. Subtract the mean from each value. 3. Square each difference. 4. Add the squares. 5. Divide by N (population) or n - 1 (sample). 6. Take the square root.

Worked example: the same six numbers both ways

Data: 4, 8, 6, 5, 3, 7. Mean = $33 \div 6 = 5.5$

Value x	x - mean	(x - mean) ²
4	-1.5	2.25
8	2.5	6.25
6	0.5	0.25
5	-0.5	0.25
3	-2.5	6.25
7	1.5	2.25
	Sum	17.5

	Population	Sample
Variance	$17.5 \div 6 = 2.917$	$17.5 \div 5 = 3.5$
Standard deviation	$\sqrt{2.917} = 1.71$	$\sqrt{3.5} = 1.87$

The sample value is always a little larger; the difference shrinks as n grows. In an exam, if the question says 'a sample of', use n - 1; if it gives the whole population, use N; if it is silent, most school courses use the population formula and say so in the formula sheet.

The other 'types' people mean

Name	What it is	Formula or rule
Variance	The standard deviation squared; the step before the square root	σ^2 or s^2
Standard error of the mean	How much a sample mean varies from sample to sample	$s \div \sqrt{n}$
Relative standard deviation (coefficient of variation)	Spread as a percentage of the mean, for comparing data in different units	$(s \div \text{mean}) \times 100\%$
Pooled standard deviation	One spread for two or more groups assumed to have the same variance	$\sqrt{[(n_1-1)s_1^2 + (n_2-1)s_2^2] \div (n_1 + n_2 - 2)}$
Weighted standard deviation	Each value counts with a weight (frequencies, grouped data)	$\sqrt{[\sum w(x - \bar{x})^2 \div \sum w]}$
Mean absolute deviation (not a standard deviation)	Average distance from the mean without squaring; grade 6 to 8 courses	$\sum x - \text{mean} \div N$

Reading a standard deviation

Small SD: values cluster near the mean. Large SD: values spread out. For a roughly bell-shaped set, about 68% of values lie within one SD of the mean, 95% within two, 99.7% within three. A test with mean 70 and SD 8: most scores sit between 62 and 78, and a 90 is two and a half SDs above the mean.

Practice

- Population SD of 2, 4, 4, 4, 5, 5, 7, 9
- Sample SD of 10, 12, 23, 23, 16, 23, 21, 16
- A sample has variance 16. Its standard deviation?
- Mean 50, SD 5: what range holds about 95% of a bell-shaped set?
- Two classes: mean 60 SD 4, and mean 60 SD 12. Which is more consistent?

Answers

- 1) mean 5, squares sum 32, $\div 8 = 4$, SD = 2 2) mean 18, squares sum 192, $\div 7 = 27.43$, $s = 5.24$ 3) 4 4) 40 to 60
5) The first (SD 4)

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