Trapezium: area ½(a + b)h, properties and types

A trapezium (US: trapezoid) has one pair of parallel sides, a and b (the bases); h is the perpendicular distance between them, never a slanting side. Area = half the sum of the parallel sides times the height = the average of the bases times the height. Bases 8 and 12, height 5: ½ x 20 x 5 = 50.

[ ] Where it comes from: a diagonal splits it into triangles ½ah and ½bh; or two copies make a parallelogram of base (a + b) and height h, and the trapezium is half

[ ] Midsegment (joins the midpoints of the legs) = (a + b) ÷ 2, parallel to the bases; area = midsegment x height

[ ] Perimeter = all four sides (the height is not a side); find a missing leg by Pythagoras from the height and the overhang

Property	Every trapezium	Isosceles	Right-angled
Parallel sides	exactly one pair	yes	yes
Angle sum	360°	360°	360°
Angles on each leg (co-interior)	add to 180°	add to 180°	90° and 90° on one leg
Legs	any	equal	one leg perpendicular to the bases (= the height)
Base angles	any	equal in pairs	two right angles
Diagonals	any	equal	not equal
Symmetry	none in general	one line	none

Shape	UK, Australia, India	US and Canada
One pair of parallel sides	trapezium	trapezoid
No parallel sides	trapezoid	trapezium

US books also disagree on inclusive (at least one pair: parallelograms count) against exclusive (exactly one pair). UK exams use the exclusive definition. Isosceles height: bases 10 and 16, legs 5: overhang (16 - 10) ÷ 2 = 3, h = √(25 - 9) = 4.

Working backwards

[ ] Height: h = 2A ÷ (a + b). Area 60, bases 7 and 13: 120 ÷ 20 = 6

[ ] A parallel side: a = 2A ÷ h - b. Area 45, height 5, base 11: 90 ÷ 5 - 11 = 7

From sciencewithshobha.com/blog/trapezium-area-formula-half-the-sum-of-the-parallel-sides-times-the-height-the-properties-the-types-isosceles-right-and-scalene-trapezium-against-trapezoid-perimeter-and-worked-examples/ · 1-on-1 online maths, KS2 to A level, Grades 3 to 12, free 30-minute first class.

Worked examples, the mistakes, and a practice set

Trapezium	Working	Answer
Bases 6 and 10, height 4	½ x 16 x 4	32 cm²
Bases 3.5 m and 6.5 m, height 2 m	½ x 10 x 2	10 m²
Right-angled, bases 9 and 15, perpendicular leg 8	½ x 24 x 8; other leg √(64 + 36) = 10	96; perimeter 42
Isosceles, bases 10 and 16, legs 5	overhang 3, h = 4, ½ x 26 x 4	52
Area 84, height 7, one base 9	2 x 84 ÷ 7 = 24 = 9 + b	b = 15
Field: bases 120 m and 80 m, 60 m apart	½ x 200 x 60	6,000 m² = 0.6 ha

Mistakes

[ ] A slanting leg used as the height
[ ] The parallel sides multiplied instead of added; the half forgotten
[ ] The height added into the perimeter; mixed units
[ ] Trapezoid on a UK paper, trapezium on a US one; assuming isosceles when not stated

Curriculum	Where
England	Year 6 recognise; KS3 area; GCSE Foundation and Higher (area, compound shapes, reverse problems; check whether your year's formula sheet gives it)
Australia	Year 7-8 area of quadrilaterals; Year 9-10 composite shapes
CBSE	Class 8 Mensuration (area with derivation); Class 9 Quadrilaterals
US	Grade 6 6.G.1 (trapezoids by decomposition); Geometry (isosceles trapezoids, midsegment)

Practice

1. Area of a trapezium with parallel sides 7 cm and 11 cm and height 6 cm?
2. Area 72 m², parallel sides 10 m and 14 m. Height?
3. Isosceles trapezium, bases 12 cm and 20 cm, legs 5 cm. Height, area, perimeter?
4. One base angle is 72°. The angle at the other end of the same leg, and why?
5. Right-angled trapezium, bases 4 and 9, height 12. Area, and the slanting leg?
6. Which is a trapezium (UK definition): a shape with two pairs of parallel sides; a shape with exactly one pair and all sides different; a square?

Answers

1. ½ x 18 x 6 = 54 cm². 2. 72 = ½ x 24 x h, h = 6 m. 3. Overhang 4, h = 3; area ½ x 32 x 3 = 48 cm²; perimeter 12 + 20 + 5 + 5 = 42 cm. 4. 108°: co-interior angles between the parallel sides add to 180°. 5. ½ x 13 x 12 = 78; leg √(144 + 25) = 13. 6. Only the second; a parallelogram and a square have two pairs.