

The square root function: $y = \sqrt{x}$, domain and range, and the transformations

x	0	1	4	9	16	25	2	10	50
$y = \sqrt{x}$	0	1	2	3	4	5	1.41	3.16	7.07

[] Starts at the origin, rises to the right, flattens as it goes, never stops rising. Domain $x \geq 0$ (no real root of a negative); range $y \geq 0$ ($\sqrt{\quad}$ means the non-negative root)

[] It is the inverse of $y = x^2$ on $x \geq 0$: half a parabola on its side. The lower half, $y = -\sqrt{x}$, is a separate function

$y = a\sqrt{x - h} + k$: starting point (h, k)

Change	Effect	Example	Start	Domain	Range
$x - h$	shift right h	$y = \sqrt{x - 3}$	(3, 0)	$x \geq 3$	$y \geq 0$
$x + h$	shift left	$y = \sqrt{x + 2}$	(-2, 0)	$x \geq -2$	$y \geq 0$
$+ k$	shift up (down) k	$y = \sqrt{x} + 4$	(0, 4)	$x \geq 0$	$y \geq 4$
$a > 1$	vertical stretch	$y = 2\sqrt{x}$	(0, 0)	$x \geq 0$	$y \geq 0$, steeper
$0 < a < 1$	compression	$y = \frac{1}{2}\sqrt{x}$	(0, 0)	$x \geq 0$	$y \geq 0$, flatter
$a < 0$	reflect in x-axis	$y = -\sqrt{x}$	(0, 0)	$x \geq 0$	$y \leq 0$
$\sqrt{-x}$	reflect in y-axis	$y = \sqrt{-x}$	(0, 0)	$x \leq 0$	$y \geq 0$
together		$y = -2\sqrt{x - 1} + 3$	(1, 3)	$x \geq 1$	$y \leq 3$

Domain starts at h ($x \geq h$; $x \leq h$ if the inside is negated); range starts at k ($y \geq k$ for $a > 0$, $y \leq k$ for $a < 0$). $\sqrt{4x} = 2\sqrt{x}$: factor the inside first.

Sketch any one in four steps: $y = -2\sqrt{x - 1} + 3$

[] Start (h, k) = (1, 3)

[] Direction: inside ($x - 1$) goes right; $a = -2$ goes down

[] Make the inside 1, 4, 9: $x = 2, 5, 10$; roots 1, 2, 3; times -2: -2, -4, -6; plus 3: $y = 1, -1, -3$. Points (2, 1), (5, -1), (10, -3)

[] Smooth curve from (1, 3), flattening. Domain $x \geq 1$, range $y \leq 3$

From

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Square root equations, the mistakes, and a practice set

- [] Isolate the root; square both sides; solve; CHECK in the original. $2\sqrt{x+3} - 4 = 6 \rightarrow \sqrt{x+3} = 5 \rightarrow x+3 = 25 \rightarrow x = 22$; check $2\sqrt{25} - 4 = 6$
- [] Extraneous solutions: $\sqrt{x+3} = x-3 \rightarrow x^2 - 7x + 6 = 0 \rightarrow x = 6$ or 1 ; $x = 1$ gives $\sqrt{4} = -2$, false. Only $x = 6$
- [] $\sqrt{x-2} = -4$ has no solution: a square root is never negative

Mistakes

- [] Range 'all real numbers' (it is $y \geq k$ or $y \leq k$)
- [] $\sqrt{x-3}$ read as a shift left (inside the function the sign reverses: right 3)
- [] $\sqrt{x}$ treated as $\pm\sqrt{x}$ (the function gives one value; the equation $x^2 = a$ has two)
- [] Skipping the check after squaring
- [] Drawing a straight line, or a curve that bends back down

Curriculum	Where
US	Algebra 1 parent function; Algebra 2 HSF-IF.C.7b, HSF-BF.B.3 transformations, HSA-REI.A.2 radical equations
England	GCSE Higher recognise $y = \sqrt{x}$ and transform graphs; A-level inverse functions, domain and range
Australia	Year 10/10A graphs and transformations; Year 11 Methods functions and inverses
CBSE	Class 11 Relations and Functions: domain and range; Class 12 inverses

Practice

- Domain, range and starting point of $y = \sqrt{x-5} + 2$?
- Sketch $y = -\sqrt{x+1}$ and name three points.
- Describe the transformations from $y = \sqrt{x}$ to $y = 3\sqrt{x-2} - 4$.
- Solve $\sqrt{2x+1} = 7$.
- Solve $\sqrt{x+6} = x$ and check for extraneous solutions.
- Why does $y = \sqrt{x}$ have no points in quadrant IV?
- Write the square root function that starts at (4, -1) and passes through (5, 1).
- What is the inverse of $y = x^2$ for $x \geq 0$, and why is the restriction needed?

Answers

- $x \geq 5$; $y \geq 2$; (5, 2).
- Start (-1, 0), right and down: (0, -1), (3, -2), (8, -3).
- Right 2, down 4, vertical stretch by 3.
- $2x+1 = 49$, $x = 24$.
- $x^2 - x - 6 = 0$, $x = 3$ or -2 ; $x = -2$ gives $\sqrt{4} = -2$, false: only $x = 3$.
- Outputs are never negative (and $x \geq 0$), so no (+, -) points.
- $y = 2\sqrt{x-4} - 1$ ($a = 2$ from (5, 1): $2 \cdot 1 - 1 = 1$).
- $y = \sqrt{x}$; the full parabola fails the horizontal line test (3 and -3 both square to 9).