

Square numbers 1 to 100 (1, 4, 9 ... 100), the squares from 1^2 to 100^2 , and the cube numbers

A square number is a whole number times itself: $7 \times 7 = 49 = 7^2$, and 49 dots fill a 7-by-7 square. The ten squares to 100 (the diagonal of the times-table grid) are for Year 5 and Grade 4 to know by heart; 11^2 to 15^2 by GCSE and Grade 8.

n	n ²	n	n ²	n	n ²	n	n ²	n	n ²
1	1	21	441	41	1,681	61	3,721	81	6,561
2	4	22	484	42	1,764	62	3,844	82	6,724
3	9	23	529	43	1,849	63	3,969	83	6,889
4	16	24	576	44	1,936	64	4,096	84	7,056
5	25	25	625	45	2,025	65	4,225	85	7,225
6	36	26	676	46	2,116	66	4,356	86	7,396
7	49	27	729	47	2,209	67	4,489	87	7,569
8	64	28	784	48	2,304	68	4,624	88	7,744
9	81	29	841	49	2,401	69	4,761	89	7,921
10	100	30	900	50	2,500	70	4,900	90	8,100
11	121	31	961	51	2,601	71	5,041	91	8,281
12	144	32	1,024	52	2,704	72	5,184	92	8,464
13	169	33	1,089	53	2,809	73	5,329	93	8,649
14	196	34	1,156	54	2,916	74	5,476	94	8,836
15	225	35	1,225	55	3,025	75	5,625	95	9,025
16	256	36	1,296	56	3,136	76	5,776	96	9,216
17	289	37	1,369	57	3,249	77	5,929	97	9,409
18	324	38	1,444	58	3,364	78	6,084	98	9,604
19	361	39	1,521	59	3,481	79	6,241	99	9,801
20	400	40	1,600	60	3,600	80	6,400	100	10,000

The cube numbers 1^3 to 20^3

n	1	2	3	4	5	6	7	8	9	10
n ³	1	8	27	64	125	216	343	512	729	1,000
n	11	12	13	14	15	16	17	18	19	20
n ³	1,331	1,728	2,197	2,744	3,375	4,096	4,913	5,832	6,859	8,000

GCSE expects 1^3 , 2^3 , 3^3 , 4^3 , 5^3 and 10^3 from memory. 1, 64 and 729 are both squares and cubes (sixth powers).

From

sciencewithshobha.com/blog/square-numbers-1-to-100-the-list-with-the-squares-to-30-and-the-cube-numbers-what-a-square-number-is-how-to-spot-one-the-patterns-and-square-roots/ · 1-on-1 online maths and science, KS2 to GCSE, Grades 3 to 8, free 30-minute first class.

How to spot a square, the patterns, square roots, and a practice set

Is it a square number?

- [] Last digit: a square ends only in 0, 1, 4, 5, 6 or 9, never 2, 3, 7 or 8 (so 1,234, 587 and 98 are out at once; the rule rules out, not in: 15 is not a square)
- [] Digital root (add the digits until one is left) of a square is 1, 4, 7 or 9 (2,025: $2 + 0 + 2 + 5 = 9$, and $45^2 = 2,025$)
- [] The sure test: the square root is a whole number ($\sqrt{196} = 14$, square; $\sqrt{200} = 14.14\dots$, not)
- [] Prime factors: every prime appears an even number of times ($144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$, square; $72 = 2^3 \times 3^2$, not)

The patterns that make the list easy

- [] Add the odd numbers: $1, 1 + 3 = 4, 1 + 3 + 5 = 9, 1 + 3 + 5 + 7 = 16$; the gaps between squares are 3, 5, 7, 9, 11 ...
- [] Next square: $(n + 1)^2 = n^2 + 2n + 1$, so $15^2 = 196 + 28 + 1 = 225$
- [] Ends in 5: multiply the tens digit by one more and write 25 after it: $35^2 = (3 \times 4)25 = 1,225$; $85^2 = 7,225$
- [] Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$, so $41^2 - 39^2 = 80 \times 2 = 160$
- [] Every square is a multiple of 4 or one more than a multiple of 4; two consecutive triangular numbers add to a square ($10 + 15 = 25$)

Square roots and Pythagoras

$\sqrt{1}$	1	$\sqrt{36}$	6	$\sqrt{121}$	11	$\sqrt{256}$	16	$\sqrt{441}$	21
$\sqrt{4}$	2	$\sqrt{49}$	7	$\sqrt{144}$	12	$\sqrt{289}$	17	$\sqrt{484}$	22
$\sqrt{9}$	3	$\sqrt{64}$	8	$\sqrt{169}$	13	$\sqrt{324}$	18	$\sqrt{529}$	23
$\sqrt{16}$	4	$\sqrt{81}$	9	$\sqrt{196}$	14	$\sqrt{361}$	19	$\sqrt{576}$	24
$\sqrt{25}$	5	$\sqrt{100}$	10	$\sqrt{225}$	15	$\sqrt{400}$	20	$\sqrt{625}$	25

$\sqrt{2} = 1.414\dots$, $\sqrt{50}$ is between 7 and 8 (7.07). Pythagoras: $3^2 + 4^2 = 25 = 5^2$, so 3-4-5 is right-angled; so are 5-12-13 and 8-15-17.

Practice

- Write all the square numbers between 20 and 150.
- Which of these are square numbers: 121, 128, 169, 180, 225, 250? Say how you know.
- What is $13^2 - 12^2$? Find it two ways.
- Which number below 100 (other than 1) is both a square and a cube?
- Work out 45^2 with the ends-in-5 trick, then check by multiplying.
- A square garden has an area of 196 m^2 . How long is each side? And if the area were 200 m^2 (to one decimal place)?

Answers

- 25, 36, 49, 64, 81, 100, 121, 144.
2. 121, 169, 225 ($11^2, 13^2, 15^2$); 128 and 250 are not ($128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, seven 2s; $250 = 2 \times 5^3$); $180 = 2^2 \times 3^2 \times 5$, the 5 is single.
3. $169 - 144 = 25$; or $(13 + 12)(13 - 12) = 25 \times 1 = 25$.
4. $64 = 8^2 = 4^3$.
5. $(4 \times 5)25 = 2,025$; $45 \times 45 = 2,025$.
6. 14 m; 14.1 m.