# Standard, vertex and factored form of a quadratic

| Form | Equation | Shows at a glance | Find the rest |
|---|---|---|---|
| Standard | $y = ax^2 + bx + c$ | y-intercept (0, c); opens up if a > 0, down if a < 0 | vertex at $x = -b/(2a)$; roots by factoring or the formula |
| Vertex | $y = a(x - h)^2 + k$ | vertex (h, k); axis x = h; max or min value k | y-intercept at x = 0; roots by solving $a(x - h)^2 + k = 0$ |
| Factored | $y = a(x - p)(x - q)$ | roots p and q | vertex at $x = (p + q)/2$; y-intercept apq |

a is the same number in every form: the coefficient of $x^2$ once expanded. $|a| > 1$ narrow, $|a| < 1$ wide, $a < 0$ opens down. One parabola three ways: $2x^2 - 8x + 6 = 2(x - 2)^2 - 2 = 2(x - 1)(x - 3)$.

## Converting

| From → to | How | Example |
|---|---|---|
| Standard → vertex | complete the square: factor a from the $x^2$ and x terms; add and subtract (half the x coefficient)$^2$ inside; write the perfect square; multiply the subtracted number by a | $3x^2 + 12x + 5 = 3(x^2 + 4x + 4 - 4) + 5 = 3(x + 2)^2 - 7$ |
| Standard → vertex (fast) | $h = -b/(2a)$, $k = f(h)$ | $h = -12/6 = -2$, $k = 12 - 24 + 5 = -7$ |
| Vertex → standard | expand the square, collect | $3(x + 2)^2 - 7 = 3x^2 + 12x + 5$ |
| Standard → factored | factor out a, factor the trinomial (or use the formula for p, q) | $2x^2 - 8x + 6 = 2(x - 1)(x - 3)$ |
| Factored → standard | expand (FOIL), multiply by a | $2(x^2 - 4x + 3)$ |
| Factored → vertex | $h = (p + q)/2$, $k = f(h)$ | $h = 2$, $k = -2$: $2(x - 2)^2 - 2$ |

## The discriminant $D = b^2 - 4ac$

| D | Roots | Graph |
|---|---|---|
| positive | two real roots | crosses the x-axis twice |
| zero | one repeated root $x = -b/2a$ | touches at the vertex |
| negative | no real roots | misses the axis |

From sciencewithshobha.com/blog/vertex-form-standard-form-and-factored-form-of-a-quadratic-function-what-each-form-shows-how-to-convert-between-them-by-completing-the-square-the-vertex-formula-and-the-discriminant-with-worked-examples-and-a-printable-sheet/ · 1-on-1 online maths and science, Grade 3 to AP and A level, free 30-minute first class.

# Worked examples and practice

| Question | Working | Answer |
|---|---|---|
| $x^2 + 6x + 2$ in vertex form | $(x^2 + 6x + 9) - 9 + 2$ | $(x + 3)^2 - 7$; vertex (-3, -7) |
| vertex of $-2x^2 + 8x - 3$ | $h = -8/(-4) = 2$; $k = -8 + 16 - 3$ | (2, 5), a maximum |
| $(x - 4)^2 - 9$ in factored form | difference of squares | $(x - 7)(x - 1)$ |
| y-intercept of $2(x - 1)(x + 5)$ | $x = 0$: $2(-1)(5)$ | -10 |
| roots of $3x^2 - 2x + 1 = 0$ | $D = 4 - 12 = -8$ | no real roots |
| max height of $h = -16t^2 + 64t + 5$ | $t = 2$; $h = -64 + 128 + 5$ | 69 at $t = 2$ |

## Practice

[ ] 1. Write $y = x^2 - 8x + 3$ in vertex form and give the vertex.

[ ] 2. Find the vertex of $y = 2x^2 + 4x - 1$ with the formula.

[ ] 3. Write $y = x^2 - 5x + 6$ in factored form; give the roots.

[ ] 4. Expand $y = -(x + 1)^2 + 4$ into standard form.

[ ] 5. Give the axis of symmetry and the minimum value of $y = 3(x - 2)^2 + 1$.

[ ] 6. How many real roots has $x^2 + 4x + 4 = 0$?

[ ] 7. A parabola has roots -2 and 6 and $a = 1$. Give the vertex.

[ ] 8. $y = 2(x - 3)^2 - 8$: find the roots.

### Answers

1) $(x - 4)^2 - 13$; (4, -13). 2) $h = -1$, $k = -3$: (-1, -3). 3) $(x - 2)(x - 3)$; 2 and 3. 4) $-x^2 - 2x + 3$. 5) $x = 2$; minimum 1. 6) $D = 0$: one repeated root, $x = -2$. 7) $h = 2$, $k = (4)(-4) = -16$: (2, -16). 8) $(x - 3)^2 = 4$, $x = 5$ or $x = 1$.

From sciencewithshobha.com/blog/vertex-form-standard-form-and-factored-form-of-a-quadratic-function-what-each-form-shows-how-to-convert-between-them-by-completing-the-square-the-vertex-formula-and-the-discriminant-with-worked-examples-and-a-printable-sheet/ · 1-on-1 online maths and science, Grade 3 to AP and A level, free 30-minute first class.