

Integrals in calculus: the rules, the two techniques, and ten practice questions

What an integral is

Definite: the area between the curve and the x-axis from a to b (below the axis counts negative), or the total accumulated from a rate. Indefinite: the family of functions $F(x) + C$ whose derivative is $f(x)$. Fundamental theorem: the integral from a to b of $f = F(b) - F(a)$ where $F' = f$.

The rules

Rule	Formula
Power ($n \neq -1$)	$\int x^n dx = x^{(n+1)}/(n+1) + C$
Constant; sum; constant multiple	$\int k dx = kx + C$; $\int (a f + b g) dx = a \int f dx + b \int g dx$
The exception	$\int 1/x dx = \ln x + C$
Exponential	$\int e^x dx = e^x + C$; $\int e^{(kx)} dx = e^{(kx)}/k + C$
Sine, cosine	$\int \cos x dx = \sin x + C$; $\int \sin x dx = -\cos x + C$
Other trig	$\int \sec^2 x dx = \tan x + C$; $\int \sec x \tan x dx = \sec x + C$
Linear inside	$\int f(ax + b) dx = F(ax + b)/a + C$
Definite	$\int$ from a to b of $f(x) dx = F(b) - F(a)$; no C

The two techniques

Technique	Undoes	Example
Substitution: $u = \text{inside}$, $du = u' dx$	The chain rule	$\int 2x (x^2 + 1)^5 dx$: $u = x^2 + 1$, $du = 2x dx$; $\int u^5 du = u^6/6 + C = (x^2 + 1)^6/6 + C$
Parts: $\int u dv = uv - \int v du$	The product rule	$\int x e^x dx$: $u = x$, $dv = e^x dx$; $= x e^x - \int e^x dx = x e^x - e^x + C$. Choose u by LIATE

The six mistakes

- [] Dropping + C on an indefinite integral
- [] Power rule on $1/x$ (it is $\ln|x|$)
- [] Not dividing by the inside's derivative: $\int e^{(3x)} dx = e^{(3x)}/3 + C$
- [] Signs: $\int \sin x = -\cos x$, $\int \cos x = \sin x$
- [] Not changing the limits when substituting in a definite integral
- [] Area below the axis subtracted when the question wants total area

Ten practice questions

- $\int (4x^3 - 6x + 2) dx$
- $\int (3/x + 2e^x) dx$
- $\int \sqrt{x} dx$
- $\int \cos 2x dx$
- $\int$ from 0 to 2 of $x^2 dx$
- $\int$ from 1 to 4 of $(2x + 1) dx$
- $\int 2x (x^2 + 3)^4 dx$
- $\int x e^{(x^2)} dx$
- $\int x \cos x dx$
- The area between $y = x$ and $y = x^2$ from 0 to 1

Answers

- $x^4 - 3x^2 + 2x + C$
- $3 \ln|x| + 2e^x + C$
- $(2/3) x^{3/2} + C$
- $(1/2) \sin 2x + C$
- $8/3$
- $[x^2 + x]$ from 1 to 4 = $20 - 2 = 18$
- $(x^2 + 3)^5/5 + C$
- $(1/2) e^{(x^2)} + C$
- $x \sin x + \cos x + C$
- $\int (x - x^2) dx$ from 0 to 1 = $1/2 - 1/3 = 1/6$

Where integrals sit

Course	Where
AP Calculus AB	Units 6 to 8, about 40% of the exam
AP Calculus BC	AB plus parts, partial fractions, improper integrals, series
A-level	Year 12 power rule and area; Year 13 techniques
IB AA	SL basics; HL techniques
CBSE Class 12	Chapters 7 and 8, about a quarter of the paper

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