

# Calculus 1: the ten topics, and the prerequisite check

| Topic                       | Skills                                                                              | Done |
|-----------------------------|-------------------------------------------------------------------------------------|------|
| 1. Limits and continuity    | Limit laws; one-sided; at infinity; 0/0 by factoring; continuity; IVT               | [ ]  |
| 2. The derivative           | From the definition; as slope; as rate; tangent lines; differentiability            | [ ]  |
| 3. The rules                | Power, product, quotient; $e^x$ , $\ln x$ , $\sin$ , $\cos$ , $\tan$ ; inverse trig | [ ]  |
| 4. The chain rule           | Composite functions; combined with every rule                                       | [ ]  |
| 5. Implicit; related rates  | $dy/dx$ from an equation; ladder, cone, shadow problems                             | [ ]  |
| 6. Curve sketching          | Critical points; first and second derivative tests; concavity; MVT; L'Hopital       | [ ]  |
| 7. Optimisation             | Max and min in context; one variable; endpoints checked                             | [ ]  |
| 8. Antiderivatives          | Rules reversed; $+C$ ; initial-value problems                                       | [ ]  |
| 9. The definite integral    | Riemann sums; area; accumulated change; properties                                  | [ ]  |
| 10. The fundamental theorem | $F(b) - F(a)$ ; accumulation functions; substitution; first areas                   | [ ]  |

## Prerequisite check (no calculator, 15 minutes)

- Factor  $x^2 - 5x + 6$  and  $x^3 - 8$
- Simplify  $(x^2 - 9)/(x - 3)$
- Write  $1/\sqrt{x}$  and  $x^2 \cdot x^{-5}$  as single powers
- Expand  $(x + h)^2 - x^2$ , divide by  $h$
- Solve  $x^2 - 3x - 10 = 0$  and  $2x + 1 > 7$
- $f(x) = 3x + 2$ ,  $g(x) = x^2$ : find  $f(g(x))$ ,  $g(f(x))$
- $\sin(\pi/6)$ ,  $\cos(\pi/3)$ ,  $\tan(\pi/4)$ ,  $\sin(\pi/2)$
- $\ln(e^3)$ ;  $e^{\ln 5}$ ;  $\log_2 8$
- Line through  $(1, 4)$  with slope  $-2$
- Sketch  $y = x^2 - 4$  with the intercepts

Answers: 1)  $(x-2)(x-3)$ ;  $(x-2)(x^2+2x+4)$  2)  $x+3$  3)  $x^{-1/2}$ ;  $x^{-3}$  4)  $2xh + h^2$ ;  $2x + h$  5) 5 or  $-2$ ;  $x > 3$  6)  $3x^2+2$ ;  $(3x+2)^2$  7)  $1/2$ ,  $1/2$ ,  $1$ ,  $1$  8)  $3$ ;  $5$ ;  $3$  9)  $y = -2x + 6$  10)  $x = \pm 2$ ,  $y = -4$ . 9-10 right: start. 6-8: a fortnight on the misses. Under 6: a month of algebra first.

## The ten-week plan

An hour a day, five days a week. Each week: read the topic (OpenStax Vol. 1 or Paul's Notes), three worked examples covered step by step, twenty problems with answers hidden, a closed-book set of five, every error in the notebook.

| Week | Topics                                    | The check                                                                               | Done |
|------|-------------------------------------------|-----------------------------------------------------------------------------------------|------|
| 1    | Limits and continuity                     | A limit by factoring; one at infinity; a piecewise continuity                           | [ ]  |
| 2    | The derivative definition; power rule     | $x^2$ from the definition; a polynomial; a tangent line                                 | [ ]  |
| 3    | Product, quotient, chain; $e^x$ , $\ln x$ | $x^2 e^x$ , $(x+1)/(x-1)$ , $(3x^2+1)^5$ , $\ln(x^2+1)$                                 | [ ]  |
| 4    | Trig derivatives; implicit                | $\sin 3x$ , $x \cos x$ ; $dy/dx$ for $x^2 + y^2 = 25$                                   | [ ]  |
| 5    | Related rates                             | Ladder, cone, two cars                                                                  | [ ]  |
| 6    | Curve sketching                           | $x^3 - 3x$ with every feature                                                           | [ ]  |
| 7    | Optimisation; MVT                         | Open box, fence, closest point                                                          | [ ]  |
| 8    | Antiderivatives; initial values           | Polynomial, $e^x$ , $1/x$ , $\cos x$ ; a function from its derivative and a point       | [ ]  |
| 9    | Definite integral; fundamental theorem    | Area under $x^2$ from 0 to 2; a total from a rate                                       | [ ]  |
| 10   | Substitution; areas; a past exam          | $\int 2x(x^2+1)^5 dx$ ; area between $y = x$ and $y = x^2$ ; an AP AB free-response set | [ ]  |

### Free resources

OpenStax Calculus Volume 1 (the textbook); Paul's Online Math Notes (clearest explanations, full solutions); Khan Academy (drilling the rules); MIT OpenCourseWare 18.01 (lectures, problem sets, exams); 3Blue1Brown Essence of Calculus (the ideas); Desmos (graph  $f$  and  $f'$  together); College Board past AP AB free-response questions.

### Where students fail

- [ ] Limits: stopping at 0/0 instead of factoring
- [ ] Chain rule: not seeing the inside function
- [ ] Related rates: substituting numbers before differentiating
- [ ] Optimisation: not reducing to one variable; ignoring endpoints
- [ ] Integrals:  $+ C$  dropped; the inside's derivative forgotten;  $\sin/\cos$  signs

From

sciencewithshobha.com/blog/how-to-learn-calculus-1-what-it-covers-the-order-the-resources-and-a-ten-week-plan-with-a-printable-checklist/