

20 hard maths questions: Grade 6 to SAT level

Try each one before turning over. The arithmetic is Grade 6 level; the difficulty is finding the idea. Suggested time: 3 minutes each for 1 to 10, 5 minutes for 11 to 20.

1. What is $1 + 2 + 3 + \dots + 100$?
2. The smallest positive whole number leaving remainder 2 when divided by 3, 3 when divided by 4, and 4 when divided by 5?
3. The last digit of 7^{2026} ?
4. A rectangle has whole-number sides, perimeter 30 and area 56. Its sides?
5. How many diagonals does a decagon have?
6. Two fair dice are rolled. Probability the total is 7?
7. A 120 m train passes a signal post in 6 s. Time to pass completely through a 180 m platform?
8. If $x + 1/x = 3$, find $x^2 + 1/x^2$ and $x^3 + 1/x^3$.
9. The angle between the hands of a clock at 3:30?
10. Twelve people each shake hands once with every other. How many handshakes?
11. A father is four times as old as his son; in 20 years he will be twice as old. Their ages now?
12. How many three-digit numbers have digits adding to 5?
13. A right-angled triangle has legs 5 and 12. Radius of its inscribed circle?
14. A paints a room in 6 hours, B in 9. Together?
15. The average of five numbers is 20; one is removed and the average of the rest is 18. The removed number?
16. 20 L of 30% acid is diluted to 20% by adding water. How much water?
17. The sequence 2, 6, 12, 20, 30, ...: the 10th term and the rule?
18. Sum of the interior angles of a 12-sided polygon? Each interior angle of a regular pentagon?
19. How many whole numbers from 1 to 1,000 are divisible by 3 or by 5?
20. (SAT) If $3x - 2y = 7$ and $x + y = 4$, what is $x - y$?

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The solutions, with the idea each one turns on

1. 5,050. Pair $1 + 100, 2 + 99 \dots$ fifty pairs of 101. [pairing; $n(n+1) \div 2$]
2. 59. Each remainder is one less than the divisor, so $N + 1$ is a multiple of 3, 4 and 5: LCM 60, $N = 59$. [look at what is missing]
3. 9. Last digits of 7^n cycle 7, 9, 3, 1; $2026 \div 4$ leaves remainder 2: the second in the cycle. [cycles]
4. 7 and 8. Sides add to 15 and multiply to 56; factor pairs of 56: only 7×8 adds to 15. [factor pairs]
5. 35. From each of 10 vertices 7 diagonals; $10 \times 7 = 70$, each counted twice. $n(n-3) \div 2$. [count then halve]
6. $1/6$. Of 36 pairs, six total 7: (1,6) (2,5) (3,4) (4,3) (5,2) (6,1). [list the sample space]
7. 15 s. Speed $120 \div 6 = 20$ m/s; to clear the platform the front travels $180 + 120 = 300$ m; $300 \div 20 = 15$. [the train's own length counts]
8. 7, then 18. Square: $x^2 + 2 + 1/x^2 = 9 \rightarrow 7$. Then $(x + 1/x)(x^2 + 1/x^2) = x^3 + 1/x^3 + (x + 1/x)$: $21 = ? + 3 \rightarrow 18$. Never solve for x . [square the equation]
9. 75° . Minute hand at 180° ; hour hand $3.5 \times 30 = 105^\circ$; difference 75° . [the hour hand moves 0.5° a minute]
10. 66. $12 \times 11 = 132$, each handshake counted twice. $n(n-1) \div 2$. [pairs counted once]
11. Son 10, father 40. $4s + 20 = 2(s + 20) \rightarrow 2s = 20$. [set up both time points]
12. 15. Hundreds digit a from 1 to 5, then $b + c = 5 - a$: $5 + 4 + 3 + 2 + 1$ ways. [organised counting]
13. 2. Hypotenuse 13; $r = (5 + 12 - 13) \div 2$, or area $30 = r \times \text{semi-perimeter}$ 15. [area = $r \times s$]
14. 3.6 hours (3 h 36 min). $1/6 + 1/9 = 5/18$ of the room per hour; $18/5$ hours. Never average the times. [rates as fractions of the job]
15. 28. Five numbers total 100; four total 72; $100 - 72$. [totals, not averages]
16. 10 L. The acid stays 6 L: $6 \div (20 + w) = 0.2 \rightarrow 20 + w = 30$. [the unchanging quantity]
17. 110; $n(n+1)$. Differences 4, 6, 8, 10 (quadratic); terms $1 \times 2, 2 \times 3, 3 \times 4 \dots$; 10×11 . [differences, then factor]
18. $1,800^\circ$ and 108° . $(n-2) \times 180$: 10×180 for 12 sides; a pentagon $540 \div 5$. [split into triangles]
19. 467. Multiples of 3: 333; of 5: 200; of 15: 66, counted twice: $333 + 200 - 66$. [inclusion-exclusion]
20. $2. y = 4 - x \rightarrow 3x - 8 + 2x = 7 \rightarrow x = 3, y = 1$. [solve the system, or spot the combination]

The ideas, collected

Pair things up; cycles and remainders; factor pairs; count from each vertex and halve; list the sample space; include the object's own length; square the equation; the hour hand moves; set up both time points; organised counting; area = $r \times \text{semi-perimeter}$; rates as fractions of the job; totals not averages; the unchanging quantity; differences then factor; split into triangles; inclusion-exclusion; combine the equations. Learn the ideas, not the problems: redo each from its idea a week later.