Graphing proportional relationships: a straight line through the origin

y = kx graphs as a straight line through (0, 0). Both tests are needed: y = 2x + 3 is straight but misses the origin (linear, not proportional); y = x² passes through the origin but curves. On the graph, k = the y-value at x = 1 = y ÷ x at any point = the slope = the unit rate.

Relationship	Graph	Proportional?
y = 3x	straight, through the origin	yes, k = 3
y = ½x	straight, through the origin, shallower	yes, k = ½
y = 2x + 3	straight, through (0, 3)	no: linear only
y = x²	curve through the origin	no: not straight
y = 5	horizontal	no

Reading the graph

[ ] (0, 0): none of one gives none of the other (no tickets, no cost). Every proportional graph starts here
[ ] (1, k): the unit rate. (1, 12) on cost against tickets: $12 a ticket; (1, 60) on distance against time: 60 miles per hour
[ ] Any clean point (x, y): k = y ÷ x. (4, 10) gives 2.5; (5, 60) gives 12
[ ] Steeper line, larger k. Two proportional lines can cross only at the origin

Deciding from a table or an equation

[ ] Table: y ÷ x the same in every row (and (0, 0) would fit). (2, 5), (4, 10), (6, 15): 2.5 each: proportional. (2, 7), (4, 11), (6, 15): ratios 3.5, 2.75, 2.5: linear (y = 2x + 3), not proportional
[ ] Equation: y = kx with nothing added. y = 4x yes; y = 4x - 1 no; y = x ÷ 3 yes (k = ⅓); y = 3 ÷ x no
[ ] Context: a $3 flag fall plus $2 a mile is linear, not proportional (two miles do not cost twice one)

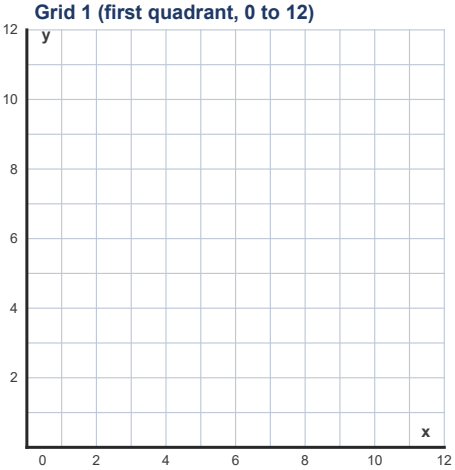
Comparing two relationships in different forms (8.EE.5)

[ ] Find k for each: graph → y at x = 1 or y ÷ x; table → y ÷ x; equation → the coefficient; words → the unit rate. Compare
[ ] Runner A's graph through (4, 32): 8 m/s. Runner B's table 3 s 21 m, 5 s 35 m: 7 m/s. A is faster, A's line is steeper
[ ] Shop P: y = 1.8x. Shop Q's graph through (5, 8): 1.6. Q is cheaper per kg

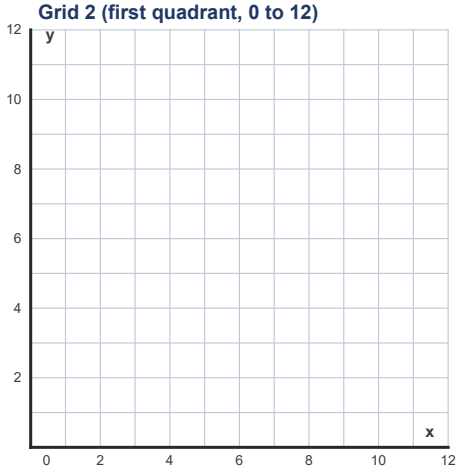
From sciencewithshobha.com/blog/graphing-proportional-relationships-what-the-graph-looks-like-through-the-origin-how-to-read-the-constant-of-proportionality-from-it-unit-rate-and-slope-and-how-to-tell-proportional-from-non-proportional/ · 1-on-1 online maths, KS2 to A level, Grades 3 to 12, free 30-minute first class.

Grids and a practice set

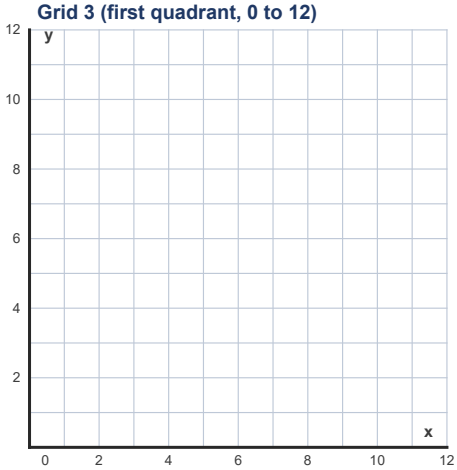
Grid 1 (first quadrant, 0 to 12)



Grid 2 (first quadrant, 0 to 12)



Grid 3 (first quadrant, 0 to 12)



Mistakes

[ ] Any straight line called proportional (it must pass through the origin)
[ ] k read at x = 0 (always 0) instead of x = 1; x ÷ y instead of y ÷ x
[ ] Axes left unlabelled: the 'what does (1, k) mean' mark lives there
[ ] The line started at the first data point instead of the origin; slopes compared by eye across different scales

Practice

1. A line passes through (0, 0) and (3, 12). Proportional? k? Slope?
2. On cost against pens, (1, 0.75) is on the line. Meaning? And (8, 6)?
3. Which show a proportional relationship: (a) a straight line through (0, 2) and (4, 10); (b) a straight line through (0, 0) and (5, 20); (c) a curve through (0, 0), (1, 1), (2, 4)?
4. Graph y = 1.5x from x = 0 to 6 on grid 1. Read y when x = 4.
5. Machine A's graph passes through (6, 90) (parts against minutes). Machine B: y = 14x. Which is faster, by how much per minute?
6. A table shows (2, 9), (4, 17), (6, 25). Proportional? Explain with the ratios and the graph.

Answers

1. Yes; k = 4; slope 4. 2. One pen costs $0.75; 8 pens cost $6. 3. (b) only. 4. y = 6. 5. A: 15 parts a minute; B: 14; A by 1 part a minute. 6. No: y ÷ x = 4.5, 4.25, 4.17 (not constant); the differences are constant (8), so it is linear, y = 4x + 1, and the line misses the origin.