Exponential growth and decay: $y = a \cdot b^t$, with $b = 1 + r$ or $1 - r$

	Growth	Decay
Formula	$y = a(1 + r)^t$	$y = a(1 - r)^t$
Factor b	1 + r, bigger than 1	1 - r, between 0 and 1
5% a period	b = 1.05	b = 0.95
20% a period	b = 1.20	b = 0.80
Doubling / halving	b = 2	b = 0.5
Graph	rises, ever steeper	falls, ever flatter, toward 0 (asymptote)

a = starting amount (t = 0); r = rate as a decimal (8% is 0.08); t = number of periods. Adding 8% is keeping 100% and adding 8%: x 1.08. Losing 15% is keeping 85%: x 0.85.

Reading a, b and r

[ ] Equation $y = 500(1.03)^t$: a = 500, b = 1.03, 3% growth. $y = 20(0.6)^t$: 40% decay

[ ] Table with equal steps: divide each value by the one before. 200, 240, 288, 345.6: ratio 1.2 → 20% growth, $y = 200(1.2)^t$. Constant DIFFERENCES (200, 240, 280) mean linear instead

[ ] Words: 'grows 2% a year' b = 1.02; 'loses 15% a year' b = 0.85; 'doubles every 20 min' b = 2 per 20 min; 'half-life 5 h' b = 0.5 per 5 h

Half-life, doubling time, compound interest, the e form

Case	Formula	Example
Half-life T	y = a·(½)^(t/T)	80 mg caffeine, T = 5 h, after 15 h: 80 x 1/8 = 10 mg; after 12 h: 80 x (½)^2.4 = 15.2 mg
Doubling time T	y = a·2^(t/T)	500 bacteria doubling every 20 min, after 2 h: 500 x 2^6 = 32,000
Compound interest, yearly	$A = P(1 + r)^t$	\$5,000 at 4% for 10 years: 5,000 x 1.04^{10} = \$7,401.22
n times a year	$A = P(1 + r/n)^{nt}$	monthly: 5,000 x $(1 + 0.04/12)^{120}$ = \$7,454.16
Continuous (Algebra 2, A-level)	$y = a \cdot e^{kt}$; $A = Pe^{rt}$	5,000 x e^0.4 = \$7,459.12; $e^k = b$; $t = \ln(y/a) \div k$
Radioactive decay	N = N0e^(-λt), λ = ln 2 ÷ T	carbon-14 T = 5,730 y: 25% left means two half-lives, 11,460 y

From sciencewithshobha.com/blog/exponential-growth-and-decay-the-formula-y-equals-a-times-1-plus-r-to-the-t-growth-and-decay-factors-doubling-time-and-half-life-worked-examples-and-practice/ · 1-on-1 online maths, KS2 to A level, Grades 3 to 12, free 30-minute first class.

Worked examples, the mistakes, and a practice set

Situation	Working	Answer
Town of 12,000 grows 2% a year; 15 years	12,000 x 1.02^{15} = 12,000 x 1.3459	16,150
Car $20,000 loses 15% a year; 5 years	20,000 x 0.85^{5} = 20,000 x 0.4437	$8,874
$5,000 at 4% yearly; 10 years	5,000 x 1.04^{10}	$7,401.22
Caffeine 80 mg, half-life 5 h; 15 h	80 x $(½)^{3}$	10 mg
Bacteria 500 doubling every 20 min; 2 h	500 x 2^{6}	32,000
Table 1,000; 900; 810; 729: after 10 periods	ratio 0.9: 1,000 x 0.9^{10}	349
Plant 10 cm to 14.4 cm in 2 weeks: weekly rate	b^{2} = 1.44, b = 1.2	20% a week
$1,000 at 6%: time to double	t = ln 2 ÷ ln 1.06	11.9 years (rule of 72: 12)

Mistakes

[ ] r used as the base: 500$(0.03)^{t}$ instead of 500$(1.03)^{t}$; the percentage left as a whole number (1 + 8)
[ ] Adding across periods instead of multiplying (20% for 3 years is x 1.2^{3} = 1.728, not +60%)
[ ] t in hours when the factor is per 5 hours (divide by 5); rounding 1.04^{10} to 1.4 (keep four decimals)
[ ] Constant differences (linear) confused with constant ratios (exponential); 'reaches zero' (it approaches zero)

Curriculum	Where
US	Algebra 1 HSF-LE.A.1/2, B.5; Algebra 2 HSF-LE.A.4 (logs) and the e form
England	GCSE Higher compound interest, depreciation, growth and decay, y = k^{x}; A-level e^{x}, ln, modelling
Australia	Year 10 compound interest and exponential graphs; Year 11-12 Methods
CBSE	Class 8 Comparing Quantities (compound interest, growth, depreciation); Class 11-12

Practice

1. Identify a, b and r; growth or decay: (a) y = 2,500$(1.07)^{t}$ (b) y = 64$(0.5)^{t}$ (c) y = 90$(0.92)^{t}$.
2. A city of 850,000 grows 1.5% a year. Population after 8 years?
3. A $1,200 laptop loses 20% a year. Value after 3 years?
4. A medicine has half-life 6 h. From 200 mg, how much after 18 h? After 15 h?
5. A table shows 50, 60, 72, 86.4 at t = 0, 1, 2, 3. Exponential? Equation?
6. $3,000 at 5% compounded yearly: formula and balance after 6 years.

Answers

1. (a) 2,500, 1.07, 7% growth; (b) 64, 0.5, 50% decay; (c) 90, 0.92, 8% decay. 2. 850,000 x 1.015^{8} = 850,000 x 1.1265 = 957,500. 3. 1,200 x 0.8^{3} = $614.40. 4. 200 x $(½)^{3}$ = 25 mg; 200 x (½)^2.5 = 35.4 mg. 5. Yes, ratio 1.2: y = 50$(1.2)^{t}$. 6. A = 3,000$(1.05)^{t}$; 3,000 x 1.3401 = $4,020.29.