

# Equations of motion (SUVAT): the five equations, where they come from, which to use, free fall

|   | SUVAT form                 | US form                             | Missing letter |
|---|----------------------------|-------------------------------------|----------------|
| 1 | $v = u + at$               | $v = v_0 + at$                      | s              |
| 2 | $s = ut + \frac{1}{2}at^2$ | $\Delta x = v_0t + \frac{1}{2}at^2$ | v              |
| 3 | $v^2 = u^2 + 2as$          | $v^2 = v_0^2 + 2a\Delta x$          | t              |
| 4 | $s = \frac{1}{2}(u + v)t$  | $\Delta x = \frac{1}{2}(v_0 + v)t$  | a              |
| 5 | $s = vt - \frac{1}{2}at^2$ | $\Delta x = vt - \frac{1}{2}at^2$   | u              |

s displacement (m), u initial velocity, v final velocity (m/s), a acceleration (m/s<sup>2</sup>), t time (s). Only for CONSTANT acceleration; split a journey into stages otherwise.

## Where they come from: the velocity-time graph (a straight line from u to v)

- [ ] gradient =  $a = (v - u)/t \rightarrow v = u + at$
- [ ] area = displacement = rectangle  $ut$  + triangle  $\frac{1}{2}(v - u)t = \frac{1}{2}at^2 \rightarrow s = ut + \frac{1}{2}at^2$
- [ ] the same area as a trapezium:  $\frac{1}{2}(u + v) \times t \rightarrow s = \frac{1}{2}(u + v)t$
- [ ] substitute  $t = (v - u)/a$  into equation 4  $\rightarrow v^2 = u^2 + 2as$
- [ ] count back from the end  $\rightarrow s = vt - \frac{1}{2}at^2$

## Which to use: the letter you neither have nor want

| You have | Want | Not involved | Use                        |
|----------|------|--------------|----------------------------|
| u, a, t  | v    | s            | $v = u + at$               |
| u, a, t  | s    | v            | $s = ut + \frac{1}{2}at^2$ |
| u, v, a  | s    | t            | $v^2 = u^2 + 2as$          |
| u, v, t  | s    | a            | $s = \frac{1}{2}(u + v)t$  |
| v, a, t  | s    | u            | $s = vt - \frac{1}{2}at^2$ |

## The sign rule and free fall

- [ ] Pick a positive direction and keep it: slowing down means negative a; thrown up with up positive,  $a = -9.8$  throughout
- [ ] Dropped ( $u = 0$ , down positive):  $v = gt$ ,  $s = \frac{1}{2}gt^2$ ,  $v^2 = 2gs$  ( $g = 9.8 \text{ m/s}^2$ ; 9.81 or 10 in some courses)
- [ ] Thrown up at  $u$ : highest point when  $v = 0$ ; time up =  $u/9.8$ ; max height =  $u^2/19.6$ ; returns at the same speed
- [ ]  $\text{km/h} \div 3.6 = \text{m/s}$ ; s is displacement (up and back = 0), not distance

From

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## Worked examples and practice

| The question                                                  | Known         | Working                            | Answer                              |
|---------------------------------------------------------------|---------------|------------------------------------|-------------------------------------|
| car from rest, $2.5 \text{ m/s}^2$ for 8 s: speed             | $u, a, t$     | $v = 0 + 2.5 \times 8$             | 20 m/s                              |
| distance in those 8 s                                         | $u, a, t$     | $\frac{1}{2} \times 2.5 \times 64$ | 80 m                                |
| car at 30 m/s brakes at $6 \text{ m/s}^2$ : stopping distance | $u, v, a$     | $0 = 900 - 12s$                    | 75 m                                |
| time to stop                                                  | $u, v, a$     | $0 = 30 - 6t$                      | 5 s                                 |
| train 10 to 25 m/s over 300 m: $a$                            | $u, v, s$     | $625 = 100 + 600a$                 | $0.875 \text{ m/s}^2$               |
| stone dropped 80 m: time, speed                               | $u = 0, s$    | $80 = 4.9t^2; v^2 = 1,568$         | 4.04 s; 39.6 m/s                    |
| ball thrown up at 20 m/s: height, time in air                 | $u, v = 0$    | $0 = 400 - 19.6s; 2 \times 20/9.8$ | 20.4 m; 4.08 s                      |
| cyclist 4 m/s, $0.5 \text{ m/s}^2$ for 12 s: distance         | $u, a, t$     | $48 + 36$                          | 84 m                                |
| lift 24 m in 6 s from rest: $a, v$                            | $s, t, u = 0$ | $24 = 18a; v = 6a$                 | $1.33 \text{ m/s}^2; 8 \text{ m/s}$ |
| ball thrown up at 15 m/s: $v$ after 2 s                       | $u, a, t$     | $15 - 19.6$                        | -4.6 m/s (down)                     |

### Practice

- [ ] 1. 5 to 25 m/s in 8 s:  $a$  and distance?  
 [ ] 2. Bus at 12 m/s decelerates at  $1.5 \text{ m/s}^2$ : distance to stop?  
 [ ] 3. From rest at  $3 \text{ m/s}^2$  over 18 m: final speed?  
 [ ] 4. Stone hits the water 2.5 s after being dropped: depth?  
 [ ] 5. Thrown up at 25 m/s: max height and time to return?  
 [ ] 6. Train 40 to 20 m/s over 500 m:  $a$  and  $t$ ?  
 [ ] 7. Rocket sled  $30 \text{ m/s}^2$  for 4 s from rest:  $v$  and  $s$ ?  
 [ ] 8. Thrown down at 5 m/s, hits at 20 m/s: height?  
 [ ] 9. Car at 72 km/h stops in 40 m: deceleration?  
 [ ] 10. Thrown up at 10 m/s: position and velocity after 1.5 s?

### Answers

- 1)  $2.5 \text{ m/s}^2$ ; 120 m 2) 48 m 3)  $10.4 \text{ m/s}$  4) 30.6 m 5)  $31.9 \text{ m/s}$ ; 5.1 s 6)  $-1.2 \text{ m/s}^2$ ; 16.7 s 7) 120 m/s; 240 m 8) 19.1 m 9) 20 m/s;  $-5 \text{ m/s}^2$   
 10) 4.0 m above;  $-4.7 \text{ m/s}$  (down)

### Sketch the velocity-time graph for question 1