

Constant of proportionality: $y = kx$, and how to find k from a table, a graph, an equation or a word problem

Two quantities are proportional when one is always the same multiple of the other: $y = k \times x$, so $k = y \div x$ for every pair. k is the constant of proportionality, also the unit rate (y when $x = 1$) and the slope of the line. 3 pencils cost \$1.50: $k = 1.50 \div 3 = 0.50$ dollars per pencil; cost = $0.50 \times$ pencils.

The four places the test puts it

Where	How to find k	Not proportional when
a table	divide every y by its x; all the answers must agree, and that answer is k	the quotients differ, or a row has $x = 0$ with y not 0
a graph	a straight line through the origin: read y at $x = 1$, or divide y by x for any point, or rise over run	the line misses the origin, or the graph curves
an equation	in $y = kx$, k is the number multiplying x (rearrange first: $2y = 6x$ is $y = 3x$)	there is an added number ($y = 4x + 2$) or a power ($y = x^2$)
a word problem	the unit rate: divide the pair given (180 miles on 6 gallons: $k = 30$ miles per gallon)	the quantities do not start together at zero

Worked examples

x	2	3	5
y	100	150	250
$y \div x$	50	50	50

$k = 50$, $y = 50x$ (miles per hour). Table $x = 1, 2, 3$ and $y = 4, 7, 10$: quotients 4, 3.5, 3.33 differ, so NOT proportional (it is $y = 3x + 2$).

Graph through (4, 12): $k = 3$. $y = 0.25x$: $k = 0.25$. $y = x \div 3$: $k = 1/3$. 5 kg of rice costs \$12: $k = \$2.40$ per kg, cost = $2.40 \times$ kg.

Graphing $y = kx$

- [] Plot (0, 0) and (1, k); draw the straight line through them; check with (2, 2k)
- [] Larger k = steeper line: $y = 3x$ sits above $y = 2x$ for every positive x
- [] To compare two relationships (one in a table, one on a graph), find each k

Inverse proportion

One goes up as the other goes down and the PRODUCT is constant: $x \times y = k$, $y = k \div x$. 4 workers take 6 days: $k = 24$ worker-days, so 8 workers take 3 days. The graph is a curve, not a line.

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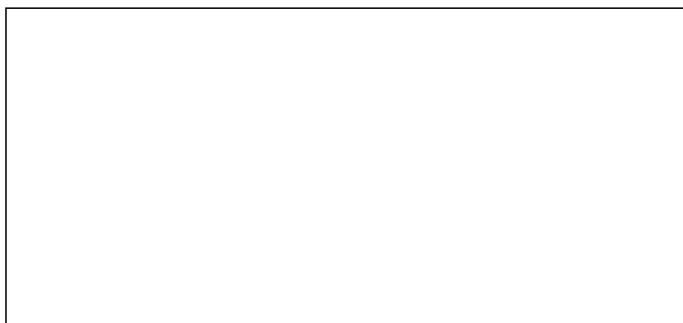
Worksheet: twelve questions

- [] 1. A table has $x = 3, 6, 9$ and $y = 12, 24, 36$. Find k and write the equation.
- [] 2. A table has $x = 1, 2, 3$ and $y = 4, 7, 10$. Is it proportional? Why?
- [] 3. A line passes through the origin and $(5, 20)$. What is k ?
- [] 4. A line passes through $(0, 3)$ and $(2, 7)$. Is it proportional?
- [] 5. What is the constant of proportionality in $y = 0.6x$?
- [] 6. Rewrite $3y = 15x$ as $y = kx$ and give k .
- [] 7. 8 notebooks cost \$20. Find k and the cost of 15 notebooks.
- [] 8. A cyclist rides 45 km in 3 hours. Write the equation and find the distance in 5 hours.
- [] 9. The point $(1, 7)$ is on the line of a proportional relationship. What is the equation?
- [] 10. Two lines through the origin: one through $(2, 10)$, the other through $(4, 12)$. Which has the larger k ?
- [] 11. 6 pumps empty a tank in 8 hours. How long do 4 pumps take?
- [] 12. A table has $x = 0, 2, 4$ and $y = 1, 5, 9$. Is it proportional?

Fill in: find k for each, then write $y = kx$

x	4	8	12	k =	equation:
y	6	12	18		
x	2	5	10	k =	equation:
y	7	17.5	35		

Draw: the graph of $y = 2.5x$ (plot $(0, 0)$, $(1, 2.5)$, $(2, 5)$, $(4, 10)$)



Answers

1) $k = 4$, $y = 4x$ 2) no: 4, 3.5, 3.33 differ 3) 4 4) no: it misses the origin 5) 0.6 6) $y = 5x$, $k = 5$ 7) \$2.50 per notebook; \$37.50 8) $d = 15t$; 75 km 9) $y = 7x$ 10) the first (5 against 3) 11) 48 pump-hours $\div 4 = 12$ hours 12) no: at $x = 0$, y should be 0. Fill in: $k = 1.5$, $y = 1.5x$; $k = 3.5$, $y = 3.5x$