# Asymptote rules for rational functions

f(x) = p(x)/q(x). Let n = degree of the top, m = degree of the bottom, a and b their leading coefficients. For large x only the highest powers matter, so f(x) behaves like (a/b)·x^(n-m): that one fact gives every rule below.

## Horizontal asymptote: compare the degrees

| Case | Horizontal asymptote | Why | Example |
|---|---|---|---|
| n < m | y = 0 | the power n - m is negative; the value shrinks to 0 | (3x + 1)/(x² - 4): y = 0 |
| n = m | y = a/b | the power is zero; the value settles at a/b | (2x² - x)/(5x² + 3): y = 2/5 |
| n > m | none | the value grows without limit | (x³ + 1)/(x - 2): none |
| n = m + 1 | none, but an oblique asymptote y = mx + c | it grows like a line: divide top by bottom, keep the quotient | (x² + 1)/(x - 1) = x + 1 + 2/(x - 1): y = x + 1 |

A graph may cross its horizontal asymptote in the middle; the asymptote describes the far left and far right only.

## Vertical asymptotes and holes

[ ] Factor top and bottom fully.
[ ] Cancel common factors. A cancelled factor (x - c) is a HOLE at x = c, not an asymptote.
[ ] Set what is left of the denominator to zero: each solution x = c is a vertical asymptote.
[ ] Example: (x² - 1)/(x² - 3x + 2) = (x - 1)(x + 1)/((x - 1)(x - 2)): hole at x = 1, vertical asymptote x = 2, horizontal y = 1.

## Other functions

| Function | Vertical | Horizontal |
|---|---|---|
| y = a·b^x + c | none | y = c |
| y = log_b(x - h) + k | x = h | none |
| y = 1/x | x = 0 | y = 0 |
| y = (ax + b)/(cx + d) | x = -d/c | y = a/c |
| y = tan x | x = π/2 + nπ | none |

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# Worked examples and practice

| Function | Horizontal / oblique | Vertical | Holes |
|---|---|---|---|
| (5x + 2)/(x² + 1) | y = 0 | none (x² + 1 ≠ 0) | none |
| (3x² + x)/(x² - 9) | y = 3 | x = 3, x = -3 | none |
| (x³ - 1)/(x + 2) | none (degrees differ by 2) | x = -2 | none |
| (2x² + 3x)/(x - 1) | oblique y = 2x + 5 | x = 1 | none |
| (x - 2)/(x² - 4) | y = 0 | x = -2 | x = 2 |
| y = 3·2^x - 4 | y = -4 | none | none |
| y = ln(x + 5) | none | x = -5 | none |
| (x² + 2x + 1)/(x + 1) | none: the graph is the line y = x + 1 | none | x = -1 |

## Practice: find every asymptote and hole

[ ] 1. (4x - 1)/(2x + 6)
[ ] 2. (x² - 4)/(x² + 1)
[ ] 3. (x² + 3x)/(x + 3)
[ ] 4. (x² - 5x)/(x - 2)
[ ] 5. (7)/(x² - 16)
[ ] 6. y = 5·3^x + 2
[ ] 7. (6x³ + 1)/(2x³ - x)
[ ] 8. y = log(x - 3)

### Answers

1) y = 2; x = -3.  2) y = 1; no vertical.  3) = x after cancelling (x + 3): hole at x = -3, no asymptotes.  4) oblique y = x - 3; x = 2.  5) y = 0; x = 4, x = -4.  6) y = 2.  7) y = 3; x = 0, x = ±1/√2.  8) x = 3.

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