

Area of a sector and arc length: a fraction of the circle

A sector is the slice between two radii; its angle θ at the centre gives the fraction of the whole circle: $\theta/360$ in degrees, $\theta/2\pi$ in radians. Area and arc are that fraction of πr^2 and $2\pi r$. Perimeter = arc + two radii. Give exact answers in terms of π first, then round to 3 s.f.

	Whole circle	Sector, θ in degrees	Sector, θ in radians
Area	πr^2	$\theta/360 \times \pi r^2$	$\frac{1}{2}r^2\theta$
Arc length	$2\pi r$	$\theta/360 \times 2\pi r$	$r\theta$
Perimeter	$2\pi r$	arc + $2r$	$r\theta + 2r$
Shortcuts		semicircle $\frac{1}{2}\pi r^2$, arc πr ; quadrant $\frac{1}{4}\pi r^2$, arc $\frac{1}{2}\pi r$	degrees $\rightarrow$ radians $\times \pi/180$

Radius	Angle	Fraction	Area	Arc	Perimeter
6 cm	60°	1/6	$6\pi = 18.8 \text{ cm}^2$	$2\pi = 6.28 \text{ cm}$	18.3 cm
10 cm	135°	3/8	$37.5\pi = 118 \text{ cm}^2$	$7.5\pi = 23.6 \text{ cm}$	43.6 cm
4 m	300°	5/6	$13.3\pi = 41.9 \text{ m}^2$	$6.67\pi = 20.9 \text{ m}$	28.9 m
9 cm	40°	1/9	$9\pi = 28.3 \text{ cm}^2$	$2\pi = 6.28 \text{ cm}$	24.3 cm
7 cm	90°	1/4	$12.25\pi = 38.5 \text{ cm}^2$	$3.5\pi = 11.0 \text{ cm}$	25.0 cm
3 cm	180°	1/2	$4.5\pi = 14.1 \text{ cm}^2$	$3\pi = 9.42 \text{ cm}$	15.4 cm
5 cm	1.2 rad		15 cm^2	6 cm	16 cm
8 cm	$\pi/3$ rad		$32\pi/3 = 33.5 \text{ cm}^2$	$8\pi/3 = 8.38 \text{ cm}$	24.4 cm

The segment (Higher, A-level)

- [] Segment = sector - the isosceles triangle on the chord: $\theta/360 \times \pi r^2 - \frac{1}{2}r^2 \sin \theta$ (degrees), or $\frac{1}{2}r^2(\theta - \sin \theta)$ (radians)
- [] Radius 6, 60°: sector 18.85, triangle $\frac{1}{2} \times 6 \times 6 \times \sin 60^\circ = 15.59$, segment 3.26 cm^2 . Chord = $2r \sin(\theta/2) = 6 \text{ cm}$

Working backwards

- [] Angle from area: $\theta = A \div (\pi r^2) \times 360$. Area 10π , $r = 5$: $0.4 \times 360 = 144^\circ$
- [] Angle from arc: $\theta = \text{arc} \div (2\pi r) \times 360$. Arc 4π , $r = 6$: 120°
- [] Radius from area and angle: $r^2 = 360A \div (\theta\pi)$. Area 24π , 60° : $r^2 = 144$, $r = 12$
- [] Radius from arc and angle: $r = 360 \times \text{arc} \div (2\pi\theta)$. Arc 5π , 150° : $r = 6$
- [] Radians: $\theta = 2A \div r^2$; $\theta = \text{arc} \div r$

From

sciencewithshobha.com/blog/area-of-a-sector-and-arc-length-the-formulas-in-degrees-and-radians-worked-examples-perimeter-of-a-sector-segments-and-practice/ · 1-on-1 online maths, KS2 to A level, Grades 3 to 12, free 30-minute first class.

The mistakes, the curriculum, and a practice set

- [] Diameter used as r ; the two radii left out of a perimeter; radii added to an area
- [] $2\pi r$ used for the area fraction or πr^2 for the arc
- [] A radian angle in the degree formula ($1.2/360 \times \pi r^2$), or 60 in $\frac{1}{2}r^2\theta$
- [] π rounded early (use the π key; leave π in to the end)
- [] Sector and segment confused; in a shaded-region question, not stating which areas are subtracted

Curriculum	Where
England	GCSE Foundation arc and sector in degrees, answers in terms of π ; Higher reverse problems and segments; A-level radians $\frac{1}{2}r^2\theta$, $r\theta$
Australia	Year 9 arc and sector; Year 10/10A composites and segments; Year 11 Methods radians
CBSE	Class 10 Areas Related to Circles: sector and segment, combinations
US	Geometry HSG-C.B.5: derive the formulas, radian measure

Practice

- Radius 12 cm, angle 45° . Area and arc length in terms of π .
- Perimeter of a sector with radius 7 cm and angle 120° , to 3 s.f.
- A sector of radius 9 cm has area 27π cm². Its angle?
- A sector of angle 72° has arc length 4π cm. Its radius?
- (Radians) Area and arc of a sector with radius 10 cm and angle 0.8 rad.
- (Higher) Area of the segment cut off by a chord subtending 90° at the centre of a circle of radius 10 cm.
- A pizza of diameter 30 cm is cut into 8 equal slices. Area of one slice, to 3 s.f.?
- A sector has perimeter 20 cm and radius 6 cm. Its arc length and angle?

Answers

- $\frac{1}{8}$ of the circle: area 18π cm², arc 3π cm.
- Arc $\frac{1}{3} \times 14\pi = 14.66$; perimeter $14.66 + 14 = 28.7$ cm.
- $27\pi \div 81\pi = 1/3$; 120° .
- $\frac{1}{5} \times 2\pi r = 4\pi$, $r = 10$ cm.
- Area $\frac{1}{2} \times 100 \times 0.8 = 40$ cm²; arc 8 cm.
- Sector $\frac{1}{4} \times 100\pi = 78.54$; triangle $\frac{1}{2} \times 10 \times 10 = 50$; segment 28.5 cm².
- $r = 15$: $\frac{1}{8} \times 225\pi = 88.4$ cm².
- Arc $20 - 12 = 8$ cm; $\theta = 8 \div (12\pi) \times 360 = 76.4^\circ$ (1.33 rad).